Decision 2 Allocation Questions

1 Five trainers, Ali, Bo, Chas, Dee and Eve, held an initial training session with each of four teams over an assault course. The completion times in minutes are recorded below.

	Ali	**Bo**	**Chas**	**Dee**	**Eve**
Team 1	16	19	18	25	24
Team 2	22	21	20	26	25
Team 3	21	22	23	21	24
Team 4	20	21	21	23	20

Each of the four teams is to be allocated a trainer and the overall time for the four teams is to be minimised. No trainer can train more than one team.

(a) Modify the table of values by adding an extra row of values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first** then rows, to decide which four trainers should be allocated to which team. State the minimum total training time for the four teams using this matching. *(8 marks)*

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2 Four of the five students Phil, Quin, Ros, Sue and Tim are to be chosen to make up a team for a mathematical relay race. The team will be asked four questions, one each on the topics A, B, C and D. A different member of the team will answer each question. Each member has to give the correct answer to the question before the next question is asked. The team with the least overall time wins.
The average times, in seconds, for each student in some practice questions are given below.

	Phil	**Quin**	**Ros**	**Sue**	**Tim**
Topic A	18	15	19	20	17
Topic B	23	24	22	25	23
Topic C	20	16	18	22	19
Topic D	21	17	18	23	20

(a) Modify the table of values by adding an extra row of values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first**, then rows, to decide which four students should be chosen for the team. State which student should be allocated to each topic and state the total time for the four students on the practice questions using this matching. *(8 marks)*

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2 Five successful applicants received the following scores when matched against suitability criteria for five jobs in a company.

	Job 1	**Job 2**	**Job 3**	**Job 4**	**Job 5**
Alex	13	11	9	10	13
Bill	15	12	12	11	12
Cath	12	10	8	14	14
Don	11	12	13	14	10
Ed	12	14	14	13	14

It is intended to allocate each applicant to a different job so as to maximise the total score of the five applicants.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $15 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table from 15. Use the Hungarian algorithm to allocate the jobs to the applicants so that the total score is maximised. *(8 marks)*

(c) It is later discovered that Bill has already been allocated to Job 4. Decide how to alter the allocation of the other jobs so as to maximise the score now possible. *(3 marks)*

2 The daily costs, in pounds, for five managers A, B, C, D and E to travel to five different centres are recorded in the table below.

	A	**B**	**C**	**D**	**E**
Centre 1	10	11	8	12	5
Centre 2	11	5	11	6	7
Centre 3	12	8	7	11	4
Centre 4	10	9	14	10	6
Centre 5	9	9	7	8	9

Using the Hungarian algorithm, each of the five managers is to be allocated to a different centre so that the overall total travel cost is minimised.

(a) By reducing the **rows first** and then the columns, show that the new table of values is

3	6	3	6	0
4	0	6	0	2
6	4	3	6	0
2	3	8	3	0
0	2	0	0	2

(3 marks)

(b) Show that the zeros in the table in part (a) can be covered with three lines and use adjustments to produce a table where five lines are required to cover the zeros.

(5 marks)

(c) Hence find the two possible ways of allocating the five managers to the five centres with the least possible total travel cost. *(3 marks)*

(d) Find the value of this minimum daily total travel cost. *(1 mark)*